Some Implications of Warping Restraint on the Behavior of Composite Anisotropic Beams

Gabriel A. Oyibo*
Polytechnic University, Farmingdale, New York

Nomenclature

a_i	= chordwise integrals
c_0', c_0	= affine space half-chord and chord, respectively
D_{ij}	= elastic constants
e	= parameter that measures the location of the reference axis relative to midchord
$\bar{h}$	= wing box depth
(h_0,α_0)	= affine space bending and torsional displacement, respectively
ℓ_0	= affine space half-span for the wing
L_0,M_0	= affine space running aerodynamic lift and moment, respectively
m_0	= affine space mass per unit span
r,L_1,L_2,D^*,D_0^*	= generic nondimensionalized stiffness parameters
$\bar{U},\bar{U}_0$	= virtual work expressions in physical and affine space, respectively
w	= displacement
(x,y,z), (x_0,y_0,z_0)	= physical and affine space coordinates, respectively
$(\Delta p, \Delta p_0)$	= differential aerodynamic pressure distriutions in physical and affine space, respectively
ρ,ρ_∞	= affine space material and air density, respectively
ω	= vibration frequency

Introduction

THE performance of modern supermaneuverable aircraft can benefit a great deal from recent significant advances in materials technology and the availability of more accurate aerodynamic prediction capabilities. Supermaneuverability as a design goal invariably calls for an optimization of the design parameters. For example, optimization may be partially accomplished by using composite materials to minimize weight. Indeed, it has been known that composite materials can be tailored to resolve the dynamic or static instability problems of these types of aircraft. The concept is referred to as aeroelastic tailoring.

While aeroelastic tailoring has tremendous advantages in the design of an aircraft, the analysis providing the basis for the aeroelastic tailoring itself is generally very involved. This is rather unfortunate since good fundamental physical insight of the tailoring mechanism is required for accurate and reliable results.

In this Note, an attempt is made to look at some dynamic theories that can be used to understand the aeroelastic tailoring mechanism. Specifically, the accuracy of the St. Venant torsion theory, which is relatively simple and frequently used in aeroelastic analysis, is examined with particular reference to the effects of the wing's aspect ratio as well as other design parameters.

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*Associate Professor, Aeronautical Engineering Department. Member AIAA.

Although earlier studies[1-3] have indicated that the St. Venant torsion theory is reasonably accurate except for aircraft wings with fairly low aspect ratios, the theory supporting that conclusion is based on the assumption that the wing is constructed of isotropic materials. Basically, the St. Venant torsion theory assumes that the rate of change of the wing's twist angle with respect to the spanwise axis is constant (for constant stiffness and torque). This assumption is hardly accurate, particularly for modern aircraft in which different construction materials are employed and the aerodynamic loads vary significantly along the wing's span. However, Refs. 1-3 have shown that, in spite of such an inaccurate assumption, the main parameter that determines the accuracy of the St. Venant theory is the wing's aspect ratio. Thus, it was determined that the theory is fairly accurate for moderate-to-high-aspect-ratio wings constructed of isotropic materials. Recent studies,[4,5] however, have shown that the conclusions of Refs. 1-3 need to be modified for wings constructed of orthotropic composite materials. Rather than using the geometric aspect ratio of the wing to determine the accuracy of St. Venant's twist theory, it is suggested that a generic stiffness ratio, as well as an effective aspect ratio that considers the wing's geometry and the ratio of the principal directional stiffness, should be considered in establishing the accuracy of St. Venant's theory.

This Note is basically an extension of the studies that were begun in Refs. 4 and 5. In this study, the task was to examine the role of coupling (elastic coupling) on the accuracy of the St. Venant theory applied to static problems. It was discovered that coupling plays a very significant role in the accuracy of St. Venant's twist theory.

Formulation

Consider an aircraft wing fabricated of composite materials and mathematically idealized as a cantilevered plate subjected to forces and moments. It can be shown that the equations of motion for such a model can be described as follows:

$$\begin{aligned} &a_1h_0^{iv} + a_2\alpha_0^{iv} + a_5\alpha_0^{iii} + \rho_0 a_1\ddot{h}_0 + \rho_0 a_2\ddot{\alpha}_0 = L_0 \\ &a_2h_0^{iv} - a_5h_0^{iii} + a_3\alpha_0^{iv} - a_4\alpha_0^{ii} + \rho_0 a_3\ddot{\alpha}_0 + \rho_0 a_2\ddot{h}_0 = M_0 \end{aligned} \tag{1}$$

where

$$a_1 = \int_{e\bar{c}_0}^{\bar{c}_0} \mathrm{d}x_0; \qquad a_2 = \int_{e\bar{c}_0}^{\bar{c}_0} x_0\,\mathrm{d}x_0$$

$$a_3 = \int_{e\bar{c}_0}^{\bar{c}_0} x_0^2\,\mathrm{d}x_0; \qquad a_4 = 2\int_{e\bar{c}_0}^{\bar{c}_0} D^*(1-\epsilon)\,\mathrm{d}x_0$$

$$L_0 = \int_{e\bar{c}_0}^{\bar{c}_0} \Delta p_0\,\mathrm{d}x_0; \quad a_5 = \int_{e\bar{c}_0}^{\bar{c}_0} L_2\,\mathrm{d}x_0$$

$$M_0 = \int_{e\bar{c}_0}^{\bar{c}_0} x_0\Delta p_0\,\mathrm{d}x_0 \tag{2}$$

$$-\infty < e < 0; \; \bar{c}_0 = \frac{c_0}{1-\epsilon}$$

$$(\;)' = \frac{\partial}{\partial y_0}; \; (\dot{\;}) = \frac{\partial}{\partial t}$$

$$\bar{U}_0 = \frac{U}{D_{22}}\left(\frac{D_{22}}{D_{11}}\right)^{1/4}; \; D^* = \frac{D_{12}+2D_{66}}{(D_{11}D_{22})^{1/2}} \tag{3}$$

$$\epsilon D^* = \frac{D_{12}}{(D_{11}D_{22})^{1/2}}$$

$$L_1 = \frac{4D_{16}}{(D_{11})^{3/4}(D_{22})^{1/4}}; \; L_2 = \frac{4D_{26}}{(D_{11})^{1/4}(D_{22})^{3/4}}$$

$$\Delta p_0 = \frac{\Delta p}{D_{22}}; \; \rho_0 = \frac{\rho\bar{h}}{D_{22}} \tag{4}$$

Evolution of Warping Parameters

The evolution of the warping parameter with which to study the aeroelastic warping constraint phenomenon for wings fabricated of composite materials is a process that depends on the sophistication of the wing's mathematical model: whether coupling effects are included, whether the wing's chordwise curvatures are included, etc. Therefore, any warping parameter is as good as the corresponding wing's displacements assumptions. However, the virtual work equation makes it possible for the analyst to determine its effective independent variables even before the displacement assumptions are made. By nondimensionalizing the spanwise space variable in Eq. (1), depending on whether one is interested in the static, dynamic, coupled, or uncoupled displacements, one of the following warping parameters may be useful:

$$\lambda_c = \frac{\ell_0}{c_0}\sqrt{(3/2)D_0^*} \tag{5}$$

$$\bar{\lambda}_c = \frac{\ell_0}{c_0}\sqrt{(3/2)[D_0^* - (L_2^2/8)]} \tag{6}$$

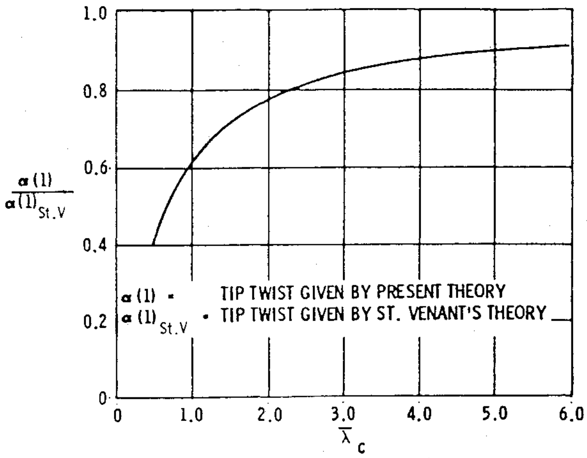

Fig. 1 Wing tip twist ratios for simple and more involved theory (distributed load and low-to-moderate coupling).

where

$$D_0^* = D^*(1-\epsilon) \tag{7}$$

(ℓ_0/c_0) is defined as the wing's effective aspect ratio and D_0^* and L are the generalized stiffness and coupling ratios, respectively (defined in earlier work such as Refs. 5 and 6).

Equations (5) and (6) represent the appropriate warping parameter for dynamic deformation and static displacement with elastic cross coupling.

It was discovered in this study that evolving the warping parameter in a manner shown in Eqs. (1-3), should enable one to effectively investigate the effects of warping on the composite wing's dynamics (or the accuracy of St. Venant's theory). From the lamination theory for composites, it is known that, while D_0^* and (ℓ_0/c_0) are always positive, L can be positive or negative. However, from Eqs. (1) and (3), it is clear that whether a composite wing has positive or negative coupling, the warping effect (in terms of $\bar{\lambda}_c$) is unchanged.

Computations

By using the evolved warping parameters defined in Eqs. (7) and (8) and appropriate boundary conditions, the boundary value problems associated with Eq. (4) are solved in a closed-form manner to determine the wing's static twist.

The wing loading conditions considered in this analysis are steady-state distributed and steady-state concentrated twist loads.

Steady Distributed Twist Loads

For a wing with a constant uniformly distributed spanwise twisting moment f_0 resulting from a steady-state coupled bending-torsion displacements, the exact closed-form solutions for the mode shape α_0, satisfying the appropriate boundary conditions, is given by

$$\alpha_0(y_0) = \frac{6f_0\ell_0^2}{c_0^3(4\bar{\lambda})^2}\left[\bar{y}_0 - \frac{\bar{y}_0^2}{2} - \frac{\sinh 4\bar{\lambda}_c\bar{y}_0}{4\bar{\lambda}_c} + \frac{1}{4\bar{\lambda}_c}\left(\tanh 4\bar{\lambda}_c + \frac{1}{4\bar{\lambda}_c\cosh 4\bar{\lambda}_c}\right)\left(\cosh 4\bar{\lambda}_c\bar{y}_0 - 1\right)\right] \tag{8}$$

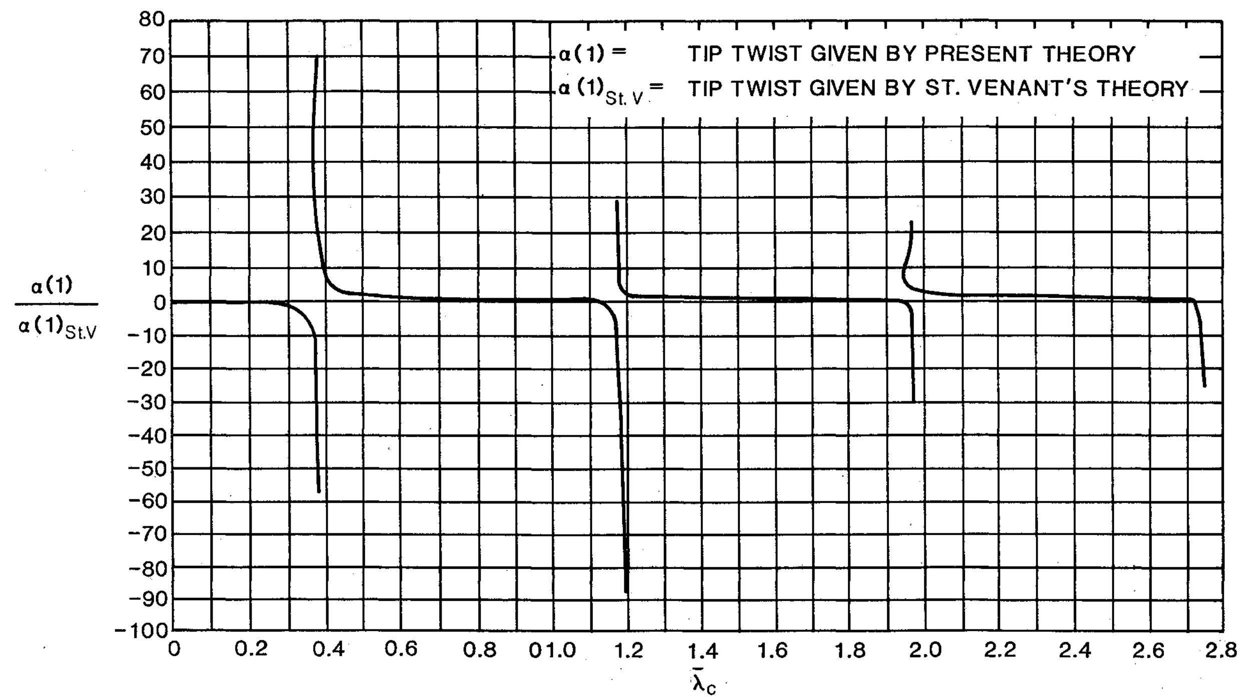

Fig. 2 Wing tip twist ratios comparing simple and more involved theory (with substantial coupling).

Equation (8) is therefore a closed-form coupled twist distribution for a composite wing with the warping effects accounted for, where

$$\bar{y}_0 = y_0/\ell_0 \tag{9}$$

When Eq. (8) is evaluated at the wing tip and compared to an equivalent expression predicted by St. Venant's theory, the following expression is obtained:

$$\frac{\alpha_0(1)}{\alpha_0(1)_{\text{St.V}}} = 1 - \frac{\tanh 4\bar{\lambda}_c}{2\bar{\lambda}_c} - \frac{1}{9\bar{\lambda}_c^2}\left(\frac{1}{\cosh 4\bar{\lambda}_c} - 1\right) \tag{10}$$

where $\alpha_0(1)$ is the wing tip twist given by Eq. (8) while $\alpha_0(1)$ is the wing tip twist given by the St. Venant torsion theory. A plot of Eq. (10) is shown in Fig. 1.

Steady-State Concentrated Tip Twist Loads

If the wing is under the influence of a concentrated twisting moment F_0 at the tip as a result of a steady-state coupled bending torsion displacement, the exact closed-form twist distribution that satisfies these equations of motion and their associated boundary conditions is given by

$$\alpha_0(\bar{y}_0) = \frac{6F_0\,\ell_0}{c_0^3(4\bar{\lambda}_c)^2}\left[\bar{y}_0 - \frac{\sinh 4\bar{\lambda}_c\bar{y}_0}{4\bar{\lambda}_c} + \frac{\tanh 4\bar{\lambda}_c}{4\bar{\lambda}_c} \times (\cosh\bar{\lambda}_c\bar{y}_0 - 1)\right] \tag{11}$$

When the twist distribution given by Eq. (11) is evaluated at the wing tip and compared to its counterpart predicted by the St. Venant torsion theory, the following expression is obtained:

$$\frac{\alpha_0(1)}{\alpha_0(1)_{\text{St.V}}} = 1 - \frac{\tanh 4\bar{\lambda}_c}{4\bar{\lambda}_c} \tag{12}$$

It should be noted that the ratio given by Eq. (12) was plotted for the real values of $\bar{\lambda}_c$ in Refs. 5 and 6 and was shown to represent conditions where any errors resulting from using St. Venant's torsion theory are conservative (overdesign rather than underdesign). In this analysis, Eq. (12) is examined when $\bar{\lambda}_c$ is imaginary, which is possible if L_2 is very large. Under such circumstances, Eq. (12) becomes

$$\frac{\alpha_0(1)}{\alpha(1)_{\text{St.V}}} = 1 - \frac{\tan 4\bar{\lambda}_c}{4\bar{\lambda}_c} \tag{13}$$

Figure 2 depicts the conditions given by Eq. (13). It is therefore seen from the figure that there are certain ranges of $\bar{\lambda}_c$ for which nonconservative errors are possible by using the St. Venant twist theory.

Results and Conclusions

The results are shown in Figs. 1 and 2. Figure 1 shows a comparison of the static wing tip twist obtained in the present study and that obtained via St. Venant's twist theory in the presence of statically distributed forces and low-to-moderate coupling. Figure 2 shows the trend for concentrated forces and substantial coupling. In Fig. 1, it is seen that the presence of coupling makes the errors of St. Venant's theory worse. This seems to suggest that the more sophisticated theory is more important for wings with coupling (e.g., wings aeroelastically tailored using elastic cross coupling).

Figure 2 also shows that nonconservative errors $\{|[\alpha_0(1)]/[\alpha_{0\text{St.V}}^{(1)}]| > 1\}$ are possible.

Using Figs. 1 and 2, the following conclusions can be summarized: 1) ignoring warping arbitrarily using St. Venant's theory could result in very significant errors (as high as over 80%) in analytical results for composite aircraft wings; 2) warping is more important (St. Venant's theory is less accurate) for wings with coupling; and 3) St. Venant's theory (which has always been shown to be conservative[1,2] can be nonconservative or St. Venant's approximation can lead to an unsafe design error (underdesign rather than overdesign from a stability point of view).

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Investigation of Internal Singularity Methods for Multielement Airfoils

D. R. Chen* and M. J. Sheu†

National Tsing Hua University, Hsinchu, Taiwan, Republic of China

I. Introduction

A NUMBER of integral equation methods are currently used for solving linear potential flows about single airfoils, e.g., Hess and Smith,[1] Bristow,[2] Hess,[3,4] Maskew and Woodward,[5] and Chen and Dalton,[6] and about multielement airfoils, e.g., Banyopadhyay et al.[7] The comparisons of the solutions of these numerical methods for single airfoils are shown in Ref. 8. The analytical solution for two-dimensional potential flow past the airfoils had been presented by Williams[9] and James.[10] The method developed by Garrick[11] can analyze flow about two-element airfoils. The method developed by Ives,[12] Halsey,[13] and Suddhoo and Hall[14] can analyze the flow about multielement airfoils.

A numerical solution, which is obtained by the internal linear-vortex-source flat-element method, is comared with the

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*Postgraduate Student, Power Mechanical Engineering Department.

†Associate Professor, Power Mechanical Engineering Department. Member AIAA.